International Journal of Theoretical Physics, Vol. 22, No. 5, 1983

Vanishing of the Cosmological Constant in Non-Abelian Kaluza–Klein Theories

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Received September 15, 1981

We present a new approach to the unification of gravity and non-Abelian gauge fields in the framework of Kaluza–Klein theory. It consists in introducing a new connection on the $(n+4)$-dimensional manifold P (metrized principal fiber bundle). This connection is metrical, but with nonvanishing torsion. An enormous cosmological term in the Einstein equations vanishes due to this connection. The new connection simultaneously cancels Planck's mass term in the Dirac equation for the five-dimensional case. The usual interpretation of geodesic equations is still valid.

1. INTRODUCTION

In this paper we propose a way of avoiding an enormous cosmological constant from the Kaluza–Klein theory for an arbitrary non-Abelian group (Cho, 1975). The classical Kaluza–Klein theory (Kaluza, 1921) unifies two major concepts in physics: (1) local coordinate invariance; (2) local gauge invariance. The first is basic for general relativity theory and the second is fundamental for electrodynamics. The Kaluza–Klein theory reduces these two concepts to the first, but in a more than four-dimensional world. In the electromagnetic case we deal with a five-dimensional manifold.

In the papers by Lichnerowicz (1955a), Rayski (1965), Tonnelat (1965), and Bergman (1942) one may find both the consecutive steps in the creation of the theory and various approaches to it. The final form of the theory was achieved in Bergman (1942). The equivalence of Kaluza and Klein's and Utiyama's (1956) theories of gauge fields was discovered by Trautman (1970). Naturally Utiyama's approach is more general and makes possible the creation of unified theories of the Yang–Mills' field and gravitation. In all these approaches the authors use a Riemannian connection on a five-dimensional manifold.

Now we know the principle of a local invariance is fundamental also for weak and strong interactions (Weinberg-Salam model, QCD), but the gauge groups are non-Abelian. Thus it seems natural and important to generalize the Kaluza–Klein procedure from the Abelian $U(1)$ group to non-Abelian groups. This is done by Kerner (1968) and Cho (1975). The authors work with a Riemannian connection on an $(n+4)$-dimensional manifold. Einstein equations with the energy momentum tensor of Yang–Mills fields and Yang–Mills equations are obtained as general results in Cho (1975) and Kerner (1968). Unfortunately these Einstein equations have a cosmological term and the cosmological constant is enormous, about $1/l_{\mathrm{Pl}}^2$, where $l_{\mathrm{Pl}} = (G\hbar/C^3)^{1/2}$ is a Planck's length. This is a pity, and may lead one to suspect that the Kaluza–Klein approach fails.

There are other obstacles. In the classical Kaluza–Klein theory (five-dimensional) there are no "interference effects" between gravitational and electromagnetic fields. W. Pauli in 1933 said that electricity and gravity were separated like oil and water in this theory. This theory reproduces (in the five-dimensional case) the well-known Einstein and Maxwell equations. But one may obtain some gravitational–electromagnetical effects if one introduces spinor fields on a five-dimensional manifold and generalize minimal coupling scheme. In this way we may obtain a new effect, the dipole electric moment of fermion. This was done first by W. Thirring (1972). But, unfortunately Thirring's results were obtained at some price. Namely, there exists an unwanted minimal fermion mass (of order 1 μg)—Planck's mass term.

Summing up, one may say that this approach fails. But the general idea is beautiful and elegant and it would be very important to avoid all these troubles. The general way is as follows: to change the geometry of the $(n+4)$-dimensional manifold to cancel the cosmological constant. This was done in Kopczynski (1979, 1980), Orzalesi and Pauri (1981), and Orzalesi (1981). In order to cancel Planck's mass term in the Dirac equation in Klein–Kaluza theory, one is forced to introduce a new kind of gauge derivative for the spinor field (Kalinowski, 1981a, b) (five-dimensional case). This new gauge derivative induces on the five-dimensional manifold a new connection. It is very easy to generalize this connection to the $(n+4)$-dimensional case. One may ask: What about the cosmological constant for such a Klein–Kaluza theory? The answer is: *It vanishes*. Thus, it seems that this connection is distinguished. Thus we avoid two basic troubles: an enormous cosmological constant in the Einstein equations and Planck's mass term in the Dirac equation. Simultaneously we get some "interference effects" between gravitational and gauge fields (electromagnetic or Yang–Mills).

The paper is organized as follows. In Section 2 we introduce all geometrical quantities and notations which we use throughout the paper. In Section 3 we deal with the Kaluza–Klein theory. We introduce the new connection $\hat{\omega}_{AB}$ on P, calculate its torsion and curvature and prove that in scalar curvature there is no cosmological constant. We also prove that the connection is metrical and derive Einsteins' and Yang–Mills' equations. It is proved that the normal interpretation of a geodesic equation (as an equation of motion for a test particle) is still valid.

2. ELEMENTS OF GEOMETRY

In this section we introduce the notations and define geometric quantities used in the paper. We use a smooth principal bundle P, which includes in its definition the following list of differentiable manifolds and smooth maps:

A total (bundle) space P.

A Lie group G-structural group.

A base space E; in our case it is a space-time.

A projection π: $P \rightarrow E$.

A map φ: $P \times G \rightarrow P$ defining the action of G on P, such that if $a, b \in G$ and $\varepsilon \in G$ is the unit element then

$$\varphi(a) \circ \varphi(b) = \varphi(ba) \quad \text{and} \quad \varphi(\varepsilon) = id \tag{1}$$

where $\varphi(a)p = \varphi(p, a)$, and moreover

$$\pi \circ \varphi(a) = \pi$$

A connection form ω on P with values in the Lie algebra of group G.

Let $\varphi'(a)$ be the tangent map to $\varphi(a)$ whereas $\varphi^*(a)$ is contragredient to $\varphi(a)$ at point a. The form ω is a form of *ad* type, i.e.,

$$\varphi^*(a)\omega = ad'_{a^{-1}}\omega \tag{2}$$

where $ad'_{a^{-1}}$ is the tangent map to the internal automorphism of the group

$$ad_a(b) = aba^{-1}$$

Due to the form ω we may introduce the distribution field of linear elements

H_r, $r \in P$, where $H_r \subset T_r(P)$ is a subspace of the space tangent to P at a point r and

$$v \in H_r \Leftrightarrow \omega_r(v) = 0 \tag{3}$$

We have

$$T_r(P) = V_r \oplus H_r \tag{4}$$

where H_r is called a subspace of horizontal vectors and V_r of vertical vectors. For vertical vectors $v \in V_r$ we have $\pi'(V) = 0$. This means that v is tangent to fibers. Let us define

$$v = \text{hor}(v) + \text{ver}(v), \qquad \text{hor}(v) \in H_r, \qquad \text{ver}(v) \in V_r \tag{5}$$

It is known that the distribution H_r is equivalent to a choice of connection ω. We use the operation "hor" for forms, i.e.,

$$(\text{hor}\,\beta)(X, Y) = \beta(\text{hor}\,X, \text{hor}\,Y) \tag{6}$$

where

$$X, Y \in T_r(P)$$

The two-form of curvature of connection is

$$\Omega = \text{hor}\, d\omega \tag{7}$$

It is also a form of "*ad*" type like ω; Ω obeys the structural Cartan equation:

$$\Omega = d\omega + \tfrac{1}{2}[\omega, \omega] \tag{8}$$

where

$$[\omega, \omega](X, Y) = [\omega(X), \omega(Y)]$$

Bianchi's identity for ω is

$$\text{hor}\, d\Omega = 0 \tag{9}$$

For the principal fiber bundle we use the convenient scheme shown in Figure 1.

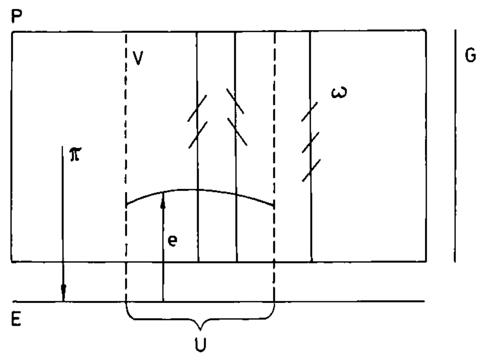

Fig. 1. Principal fiber bundle P.

The map e: $E \supset U \to P$, so that $e \circ \pi = id$ is called a cross section. From the physical point of view this means choosing a gauge. Thus

$$e^*\omega = e^*(\omega^a X_a) + A^a_\mu \bar{\theta}^\mu X_a$$

$$e^*\Omega = e^*(\Omega^a X_a) = \tfrac{1}{2} F^a_{\mu\nu} \bar{\theta}^\mu \wedge \bar{\theta}^\nu X_a \tag{10}$$

Further we introduce the notation

$$\Omega^a = \tfrac{1}{2} H^a_{\mu\nu} \theta^\mu \wedge \theta^\nu, \tag{11}$$

where $\theta^\mu = \pi^*(\bar{\theta}^\mu)$ and

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + C^a_{bc} A^b_\mu A^c_\nu$$

The X_a, $a = 1, 2, \ldots,$ $\dim G = n$ are generators of Lie algebra of group G and $[X_a, X_b] = C^c_{ab} X_c$. A covariant derivation d_1 on P is defined as follows:

$$d_1\Psi = \operatorname{hor} d\Psi \tag{12}$$

This derivation is called the "gauge" derivation, where Ψ is for example a spinor field on P. For a principal fiber bundle P it is possible to introduce a natural metrization in the following way:

$$\gamma(X, Y) = g(\pi' X, \pi' Y) + \lambda^2 \cdot h(\omega(X), \omega(Y)) \tag{13}$$

$$X, Y \in T(P), \qquad 0 < \lambda = \text{const}$$

where h is a Killing tensor on a group G. It is obvious that G must be semisimple. We have $h_{ab} = C^c_{ad}C^d_{bc}$, where C^c_{ab} are structural constants of Lie's algebra of group G. The formula (13) was proposed by A. Trautman (1970). The tensor γ is b-invariant with respect to group G.

In this paper we will use also a linear connection on manifolds P and E using the formalism of differential forms. So the basic quantity is a one-form of connection ω_{AB}. The two-form of curvature is

$$\Omega^A_B = d\omega^A_B + \omega^A_C \wedge \omega^C_B \tag{14}$$

and the two-form of torsion is

$$\Theta^A = D\theta^A \tag{15}$$

where θ^A are basic forms, and D means exterior covariant derivation with respect to ω_{AB}. The following relations define the interrelation between our symbols and generally used ones:

$$\begin{aligned} \omega^A_B &= \Gamma^A_{\cdot BC}\theta^C \\ \Theta^A &= \tfrac{1}{2}Q^A_{\cdot BC}\theta^B \wedge \theta^C \\ \Omega^A_{\cdot B} &= \tfrac{1}{2}R^A_{\cdot BCD}\theta^C \wedge \theta^D \end{aligned} \tag{16}$$

where Γ^A_{BC} are connection coefficients (they do not have to be symmetrical in indices B and C), $R^A_{\cdot BCD}$ is a curvature tensor, and $Q^A_{\cdot BC}$ is a torsion tensor. Covariant exterior derivation with respect to ω^A_B is given by

$$\begin{aligned} D\Xi^A &= d\Xi^A + \omega^A_{\cdot C}\Xi^C \\ D\Sigma^A_{\cdot B} &= d\Sigma^A_{\cdot B} + \omega^A_{\cdot C} \wedge \Sigma^C_{\cdot B} - \omega^C_{\cdot B} \wedge \Sigma^A_{\cdot C} \end{aligned} \tag{17}$$

The forms of curvature $\Omega^A_{\cdot B}$ and torsion Θ^A obey Bianchi's identities

$$\begin{aligned} D\Omega^A_{\cdot B} &= 0 \\ D\Theta^A &= \Omega^A_{\cdot B} \wedge \theta^B \end{aligned} \tag{18}$$

All quantities introduced in this paper and their precise definitions can be found in the papers by Kobayashi and Nomizu (1963), Trautman (1970, 1971, 1980), and Lichnerowicz (1955b).

3. THE KLEIN–KALUZA THEORY

Let us introduce the principal fiber bundle P over the space-time E with the structural group G and with the projection π. Now we turn to the metrization of bundle P. Let us suppose that (E, g) is a manifold with a metric tensor g and Riemannian connection $\overline{\omega}_{\alpha\beta}$, where $g = g_{\alpha\beta}\overline{\theta}^{\alpha}\otimes\overline{\theta}^{\beta}$. The signature of g is $(- - - +)$ and $\overline{\theta}^{\alpha}$ is a frame on E. Let us introduce natural frame on P:

$$\theta^A = \left(\pi^*(\overline{\theta}^{\alpha}), \theta^a = \lambda\omega^a\right), \qquad \lambda > 0, \text{const} \tag{19}$$

$\omega = \omega^a X_a$ is a connection on P. It is convenient to introduce the following notations. Capital Latin indices A, B, C run $1, 2, 3, 4\ldots, n+4$, $\dim G = n$. Lower case Greek indices $\alpha, \beta, \gamma, \delta = 1, 2, 3, 4$, and lower case Latin indices $a, b, c, d = 5, 6, \ldots, n+4$. The symbol "-" over θ^{α} and $\omega_{\alpha\beta}$ (i.e., $\overline{\theta}^{\alpha}$, $\overline{\omega}_{\alpha\beta}$) indicates that both quantities are defined on E.

Let us introduce now a tensor $\gamma = \gamma_{AB}\theta^A\otimes\theta^B$ on the manifold P in the natural way (Trautman, 1970). Let $X, Y \in T_{\tan}(P)$. Thus according to formula (13) we have

$$\gamma(X, Y) = g(\pi' X, \pi' Y) + h_{ab}\theta^a(X)\theta^b(Y)$$

or

$$\gamma = \pi^* g + h_{ab}\theta^a\otimes\theta^b \tag{20}$$

Tensor γ has signature

$$(- - - + \underbrace{- - \cdots -}_{n \text{ times}})$$

In this particular frame this tensor has a form

$$\gamma_{AB} = \left(\begin{array}{c|c} g_{\alpha\beta} & 0 \\ \hline 0 & h_{ab} \end{array}\right) \tag{21}$$

It is clear that the frame θ^A is partially nonholonomic, because

$$d\theta^a = \lambda\left(\Omega^a - \frac{1}{2\lambda^2}C^a_{bc}\theta^b \wedge \theta^c\right) \neq 0 \tag{22}$$

We also introduce a dual frame

$$\gamma(\xi_A) = \gamma_{AB}\theta^B \tag{23}$$

We have $\xi_A = (\xi_\alpha, \xi_a)$ and according to Section 2

$$\underset{\xi_a}{\mathfrak{L}}\gamma = 0 \tag{24}$$

Thus ξ_a are Killing vectors of the metric γ. Let us define now Riemannian connection ω_{AB} on P and exterior covariant derivative D with respect to ω_{AB} such that

$$D\gamma_{AB} = 0 \quad \text{and} \quad D\theta^A = 0 \tag{25}$$

The solution of (25) is

$$\begin{aligned}
\omega_{\alpha\beta} &= \pi^*(\bar{\omega}_{\alpha\beta}) - \frac{\lambda}{2} H_{\alpha\beta a}\theta^a \\
\omega_{\alpha b} &= -\omega_{b\alpha} = -\frac{\lambda}{2} H_{\alpha\gamma b}\theta^\gamma \\
\omega_{ab} &= -\omega_{ba} = -\frac{1}{2\lambda} C_{abc}\theta^c
\end{aligned} \tag{26}$$

ω_{AB} is invariant with respect to an action of group G (Cho, 1975; Kopczyński, 1980). In the papers Kalinowski (1981a,b) we introduce a new kind of gauge derivative for spinor field Ψ. Due to the derivative we avoid some troubles which appear in Thirring (1972). We get for the electromagnetic case $[G = U(1)]$ the dipole electric moment of the fermion without Planck's mass term in Dirac equation. Now we recall a definition of D:

$$\mathfrak{D}\Psi = \operatorname{hor} D\Psi = d_1\Psi + \operatorname{hor}(\omega^{AB})\hat{\sigma}_{AB}\Psi \tag{27}$$

It is easy to see that in this case we work rather with the connection

$$\hat{\omega}_{AB} = \operatorname{hor}(\omega_{AB}) \tag{28}$$

than ω_{AB}.

It would be interesting to examine this connection in the framework of the Klein–Kaluza theory. $\hat{\omega}_{AB}$ is invariant with respect to an action of group G, because the group action of G does not mix horizontal and vertical parts. We proceed as follows. First we calculate the connection coefficients $\hat{\omega}_{AB}$. Then we calculate curvature forms, the Ricci tensor and the curvature scalar. From the Hilbert variational principle with respect to $g_{\alpha\beta}$ and the connection of bundle P we get gravitational and Yang–Mills equations. One obtains

$$\begin{aligned}
\hat{\omega}_{\alpha\beta} &= \pi^*(\bar{\omega}_{\alpha\beta}) \\
\hat{\omega}_{\alpha b} &= -\hat{\omega}_{b\alpha} = -\frac{\lambda}{2} H_{\alpha\gamma b}\theta^{\gamma} \\
\hat{\omega}_{ab} &= \hat{\omega}_{ba} = 0
\end{aligned} \tag{29}$$

It is easy to check that $\hat{\omega}_{AB}$ is metrical, but with nonvanishing torsion

$$\hat{D}\gamma_{AB} = 0, \qquad \hat{\Theta}^A = \hat{D}\theta^A \neq 0 \tag{30}$$

From (29) and (30) we get

$$\begin{aligned}
\hat{\Theta}^{\alpha} &= -\frac{\lambda}{2} H^{\alpha}_{\cdot\gamma b}\theta^{\gamma} \wedge \theta^{b} \\
\hat{\Theta}^{a} &= -\frac{1}{2\lambda} C^{a}_{\cdot bc}\theta^{b} \wedge \theta^{c}
\end{aligned} \tag{31}$$

The torsion is a nonhorizontal form (horizontality is understood in the sense of the connection ω of bundle P). Now we calculate curvature two-forms for $\hat{\omega}_{AB}$,

$$\hat{\Omega}^{\alpha}_{\cdot\beta} = \pi^*(\bar{\Omega}^{\alpha}_{\cdot\beta}) - \frac{\lambda^2}{4} h_{ab} H^{\alpha}_{\cdot[\gamma}{}^{a} H_{|\beta|\delta]\cdot}{}^{b}\theta^{\gamma} \wedge \theta^{\delta} \tag{32}$$

where $\bar{\Omega}^{\alpha}_{\cdot\beta}$ is a curvature two-form for $\bar{\omega}^{\alpha}_{\beta}$:

$$\hat{\Omega}^{\alpha}_{\cdot b} = -\frac{\lambda}{2} \overset{\text{gauge}}{\overline{\nabla}}_{[\delta} H^{\alpha}_{\cdot\gamma]b}\theta^{\gamma} \wedge \theta^{\delta} - \frac{1}{2} C^{d}_{\cdot ba} H^{\alpha}_{\cdot\gamma d}\theta^{\gamma} \wedge \theta^{a}, \tag{33}$$

where $\overset{\text{gauge}}{\overline{\nabla}}_{\delta} H^{\alpha}_{\cdot\gamma}$ is a gauge (with respect to ω) *and* generally covariant derivative with respect to the connection $\bar{\omega}^{\alpha}_{\beta}$ on E

$$\hat{\Omega}^{a}_{\cdot b} = -\frac{\lambda^2}{4} H_{\gamma[\delta}{}^{a} H^{\gamma}_{\cdot\rho]b}\theta^{\delta} \wedge \theta^{\rho} \tag{34}$$

After some calculations one gets the curvature tensor $\hat{R}_{ABCD}$ and the Ricci tensor. Then one obtain the curvature scalar $\hat{R}$,

$$\hat{R}_{\alpha\beta} = \hat{R}^{A}_{\alpha A\beta} = \overline{R}_{\alpha\beta} + \frac{\lambda^2}{4} h_{ab} H^{\gamma a}_{\cdot\beta\cdot} H_{\alpha\gamma\cdot}{}^{b}$$

$$\hat{R}_{\alpha a} = 0 \tag{35}$$

$$\hat{R}_{a\alpha} = \frac{\lambda}{2} \overset{\text{gauge}}{\overline{\nabla}_{\gamma}} H^{\gamma}_{\cdot\alpha a}$$

$$\hat{R}_{ab} = 0$$

and

$$\hat{R} = \hat{R}^{\cdot}_{A} A = \overline{R} - \frac{\lambda^2}{4} H^2 = \overline{R} - \frac{\lambda^2}{4} F^2 \tag{36}$$

where $H^2 = h_{ab} H^a_{\mu\nu} H^{\mu\nu b} = F^2 = h_{ab} F^a_{\mu\nu} F^{\mu\nu b}$. Thus we see we get $\hat{R}$ as a sum of two Lagrangians: $\overline{R}$ (for a gravitational field) and $-(\lambda^2/4)F^2$ (for Yang–Mills' field). In (36) there is no a cosmological constant. From the variational principle for $\hat{R}$ with respect to $g_{\alpha\beta}$ and A^a_{μ}

$$\delta \int_{v} \hat{R} (\gamma)^{1/2} d^{n+4}x = \delta \int_{u} \left(\overline{R} - \frac{\lambda^2}{4} F^2 \right) (-g)^{1/2} d^4x \int_{G} (+h)^{1/2} d^n x = 0 \tag{37}$$

where

$$\gamma = \det \gamma_{AB} = \det g_{\alpha\beta} \cdot \det h_{ab} = g \cdot h, \qquad V = U \times G$$

We get the Einstein equations and Yang–Mills equations

$$\overline{R}_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = \frac{8\pi G}{C^4} T_{\alpha\beta}$$

$$\overset{\text{gauge}}{\overline{\nabla}_{\gamma}} F^{\gamma a}_{\cdot\alpha} = 0 \tag{38}$$

where

$$T_{\alpha\beta} = \frac{1}{4\pi} h_{ab} \left(F^{\mu\, a}_{\cdot\alpha\cdot} F^{\cdot\cdot b}_{\mu\beta} - \frac{1}{4} g_{\alpha\beta} F^{\mu\nu a} F_{\mu\nu}{}^{b} \right)$$

and we put,

$$\lambda = 2\sqrt{G}/C^2$$

Now we interpret an equation of geodesic line for the connection $\hat{\omega}_{AB}$. We have

$$U^B \hat{\nabla}_B U^A = 0 \tag{39}$$

where $U^A(\tau)$ is a vector such as $g_{\alpha\beta}U^\alpha U^\beta = 1$ and U^b is of "*ad*" type. From (39) and (29) one easily gets

$$\frac{\bar{D}U^\alpha}{d\tau} - \frac{\lambda}{2} U^b H^{\alpha}_{\cdot\beta b} U^\beta = 0$$

$$\frac{dU^b}{d\tau} = 0 \tag{40}$$

where $\bar{D}/d\tau$ is a covariant derivation along the a line to which U^α is tangent. The first equation of (40) is an equation of motion of a matter point of $q_b/m_0 = \lambda U^b/2$ in both gravitational and Yang–Mills' field [q^b is a color (isotopic) charge, and m_0 is a rest mass]. The second equation of (40) means constancy of q_i/m_0 along the world line of a particle. In the case $G = U(1)$ (electromagnetic) the first equation is a classical equation for a charged particle moving in gravitational and electromagnetic fields (Lorentz's force). In the general case the equation is called Wong's equation (Kerner, 1968). Thus the usual interpretation of the geodesic line equation in the Klein–Kaluza theory is valid in our approach.

4. CONCLUSION

The connection $\hat{\omega}^A_{\cdot B}$ seems to be distinguished. Due to this connection we cancel the enormous cosmological term in the Einstein equations for non-Abelian Kaluza–Klein theory. Simultaneously the same connection cancels Planck's mass term in the Dirac equation on a five-dimensional manifold without losing "interference effects" between gravitation and electromagnetism—i.e., the dipole electric moment of the fermion. It obviously cancels such terms in the Dirac equation on an $(n+4)$-dimensional manifold and one may obtain some "interference effects" between gravitation and the Yang–Mills field. But interpretation of these new terms remains to be found. They are analogous to dipole moments of the fermion for the "electric part" of the Yang–Mills fields. The usual interpretation of the geodesic line equation is still valid. $\hat{\omega}_{AB}$ is metrical and invariant with respect to an action of the group G.

ACKNOWLEDGMENTS

I thank Professor A. Trautman and Drs. W. Kopczyński and J. Tafel for their interest and helpful discussions.

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